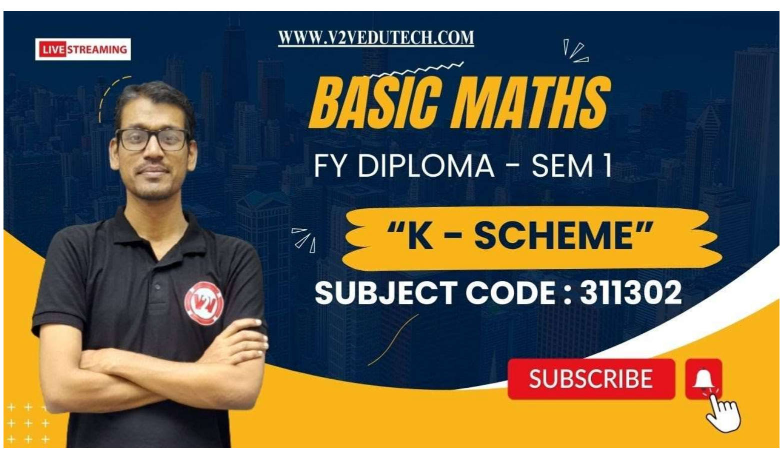
YouTube Lecture Links & Notes:

Lecture 1: Derivative: https://www.youtube.com/Derivative Lec 1

Lecture 2: Derivative: https://www.youtube.com/Derivative Lec 2

Join Whtassp Group

K – Scheme : https://chat.whatsapp.com/B5tS6rgj5pp4lRFHAWbc3P



Derivative

$$\frac{d}{dx}\,\text{const} = 0$$

$$\frac{d}{dx}x^n = n\cdot x^{n-1}$$

$$\frac{d}{dx}kx^n = k\cdot\frac{d}{dx}x^n$$

$$\frac{d}{dx}a^x = a^x\cdot\log_e a$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\log x = \frac{1}{x}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}\sec x = \sec x\cdot\tan x$$

$$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x\cdot\cot x$$

⊛ Product Rule

$$\frac{d}{dx}u\cdot v = u\cdot\frac{d}{dx}v + v\cdot\frac{d}{dx}u$$

⊛ Division Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\cdot\frac{d}{dx}u - u\cdot\frac{d}{dx}v}{v^2}$$

⊛ $$\frac{d}{dx}(u \pm v \mp w) = \frac{d}{dx}u \pm \frac{d}{dx}v \mp \frac{d}{dx}w$$

Chain Rule

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\operatorname{cosec}^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$$

Q. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ find $\frac{dy}{dx}$

$\Rightarrow$ $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

assuming $x = \tan\theta$

$x = \tan\theta$

$\tan^{-1}(x) = \theta$

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$y = 2\tan^{-1}x$$

differentiating w.r.t x

$$\frac{d}{dx} y = \frac{d}{dx} 2 \tan^{-1}x$$

$$= 2 \cdot \frac{d}{dx} \tan^{-1}x$$

$$\frac{dy}{dx} = 2\left(\frac{1}{1+x^2}\right) //$$

$$\sin 2\theta = 2\sin\theta \cdot \cos\theta$$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

if $y = (x^2+1)^{10}$ find $\frac{dy}{dx}$

$\Rightarrow$ $y = (x^2+1)^{10}$ X

$y = X^{10}$

differentiating w.r.t 'x'

$$\frac{d}{dx} y = \frac{d}{dx} X^{10}$$

$$= 10 \cdot X^{10-1} \cdot \frac{d}{dx} X$$

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

$$= 10 \cdot (x^2+1)^9 \cdot \frac{d}{dx}(x^2+1)$$

$$= 10 (x^2+1)^9 \left[\frac{d}{dx} x^2 + \frac{d}{dx} 1 \right]$$

$$= 10 (x^2+1)^9 [2x+0]$$

$$\frac{dy}{dx} = 10 (x^2+1)^9 (2x) //$$

$$= 20x (x^2+1)^9 //$$

Explicit function

$y = x^2 + 4x + 9$

$f(x) = x^2 + 4x + 9$

Implicit function

$x^2 + 4xy + y^2 = 6$

$f(x, y) = 0$

$y^2 = 4x^2 + 6xy + 7$ find $\frac{dy}{dx}$

$y^2 = 4x^2 + 6xy + 7$

differiating w.r.t 'x'

$$\frac{d}{dx} y^2 = \frac{d}{dx} 4x^2 + \frac{d}{dx} 6xy + \frac{d}{dx} 7$$

$$\frac{d}{dx} X^2 = 4 \cdot \frac{d}{dx} x^2 + 6 \cdot \frac{d}{dx} \underbrace{x}_{u}\underbrace{y}_{v} + 0$$

$$2X \cdot \frac{d}{dx} X = 4(2x) + 6\left[u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u\right]$$

$$2y \cdot \frac{d}{dx} y = 8x + 6\left[x \cdot \frac{d}{dx} y + y \frac{d}{dx} x\right]$$

$$2y \cdot \frac{dy}{dx} = 8x + \left[6x \frac{dy}{dx} + 6y\right]$$

$$2y \frac{dy}{dx} - 6x \cdot \frac{dy}{dx} = 8x + 6y$$

$$\frac{dy}{dx}(2y - 6x) = 8x + 6y$$

$$\frac{dy}{dx} = \frac{(8x+6y)}{(2y-6x)}$$

//

$y = x^{\tan x}$

(Variable)$^{\text{const}}$ $\frac{d}{dx} x^n = n \cdot x^{n-1}$

$y = x^X$ (variable)$^{\text{variable}}$

(const)$^{\text{variable}}$ $\Rightarrow \frac{d}{dx} a^x = a^x \cdot \log a$

taking log on both sides

$\log y = \log x^X$

$\log a^x = x \cdot \log a$

$\log y = X \cdot \log x$

$\log y = \tan x \cdot \log x$

diff. w.r.t 'x'

$$\frac{d}{dx} \log y = \frac{d}{dx} \underbrace{(\tan x)}_{u} \underbrace{(\log x)}_{v}$$

$$\frac{d}{dx} \log X = u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u$$

$$\frac{1}{X} \cdot \frac{d}{dx} X = \tan x \cdot \frac{d}{dx} \log x + \log x \cdot \frac{d}{dx} \tan x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \tan x \cdot \frac{1}{x} + \log x \cdot \sec^2 x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x} \cdot \tan x + \log x \cdot \sec^2 x$$

$$\frac{dy}{dx} = y\left[\frac{1}{x} \cdot \tan x + \log x \cdot \sec^2 x\right]$$

Q $y = x^{2a} - 2a^{2a} + a^{2a} + 2x^{2a}$

$\Rightarrow$ diff. w.r.t 'x'

Variable - x, y, z

$$\frac{d}{dx} y = \frac{d}{dx}\left(x^{2a} - 2a^{2a} + a^{2a} + 2x^{2a}\right)$$

$$= \frac{d}{dx} x^{2a} - \frac{d}{dx} 2a^{2a} + \frac{d}{dx} a^{2a} + \frac{d}{dx} 2x^{2a}$$

$$= (2a) \cdot x^{(2a-1)} - 0 + 0 + 2 \cdot \frac{d}{dx} x^{2a}$$

$$\frac{dy}{dx} = 2a \cdot x^{(2a-1)} + 2 \cdot (2a) \cdot x^{(2a-1)}$$

$$= 2a \cdot x^{(2a-1)} + 4a \cdot x^{(2a-1)} \quad //$$

$$= x^{(2a-1)} \left[2a + 4a\right]$$

$$\frac{dy}{dx} = x^{(2a-1)} \cdot (6a) \quad //$$

$x^2 + y^2 = 4xy$ find $\left(\frac{dy}{dx}\right)$ at $(2, -1)$ //

x_1 y_1

$\Rightarrow$ $x^2 + y^2 = 4xy$

diff. w.r.t 'x'

$$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = \frac{d}{dx} 4xy$$

u . v

$$2x + 2y \cdot \frac{dy}{dx} = 4 \cdot \frac{d}{dx} (x)(y)$$

$$2x + 2y \cdot \frac{dy}{dx} = 4\left[u \cdot \frac{d}{dx} v + v \cdot \frac{d}{dx} u\right]$$

$$2x + 2y \cdot \frac{dy}{dx} = 4\left[x \cdot \frac{dy}{dx} + y \frac{d}{dx} x\right]$$

$$2x + 2y \cdot \frac{dy}{dx} = 4x \cdot \frac{dy}{dx} + 4y.$$

$$2y \cdot \frac{dy}{dx} - 4x \frac{dy}{dx} = 4y - 2x$$

$$\frac{dy}{dx}(2y - 4x) = 4y - 2x$$

$$\frac{dy}{dx} = \frac{4y - 2x}{2y - 4x}$$

$$\left.\frac{dy}{dx}\right|_{at\ (2,-1)} = \frac{4(-1) - 2 \times 2}{2(-1) - 4 \times 2}$$

x y

$$= \frac{-8}{-10}$$

$$\left.\frac{dy}{dx}\right|_{at\ (2,-1)} = 0.8 \quad //$$

$$y = \cos^{-1}(4x^3 - 3x) \qquad \text{find } \frac{dy}{dx}$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$x = \cos\theta$$
$$\cos^{-1}x = \theta$$

$$y = \cos^{-1}\left[4\cos^3\theta - 3\cos\theta\right]$$
$$= \cos^{-1}[\cos 3\theta]$$
$$y = 3\theta$$
$$y = 3\cos^{-1}x$$

diff. w.r.t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}\,3\cos^{-1}x$$
$$= 3 \cdot \frac{d}{dx}\cos^{-1}x$$
$$\frac{dy}{dx} = 3\left(\frac{-1}{\sqrt{1-x^2}}\right)$$ //

$$x = a\cdot\cos^3\theta \quad \& \quad y = a\cdot\sin^3\theta$$

find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$

$\pi = 180°$

$\frac{\pi}{4} = \frac{180}{4}$

$\theta = 45°$ //

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$y = a\cdot\sin^3\theta$

diff. w.r.t θ

$$\frac{d}{d\theta}y = \frac{d}{d\theta}\,a\,\sin^3\theta$$
$$= a\cdot\frac{d}{d\theta}\,\sin^3\theta$$
$$= a\cdot\frac{d}{d\theta}\,x^3$$
$$= a\left[3x^{3-1}\cdot\frac{d}{d\theta}x\right]$$
$$= a\left[3(\sin\theta)^2\,\frac{d}{d\theta}\sin\theta\right]$$
$$\frac{dy}{d\theta} = a\left[3(\sin\theta)^2\cdot\cos\theta\right]$$

$x = a\cos^3\theta$

diff. w.r.t θ

$$\frac{d}{d\theta}\cdot x = \frac{d}{d\theta}\,a\,\cos^3\theta$$
$$= a\cdot\frac{d}{d\theta}\,\cos^3\theta$$
$$= a\cdot\frac{d}{d\theta}\,x^3$$
$$\frac{dx}{d\theta} = a\left[3x^{3-1}\cdot\frac{d}{d\theta}x\right]$$
$$= a\left[3(\cos\theta)^2\cdot\frac{d}{d\theta}\cos\theta\right]$$
$$\frac{dx}{d\theta} = a\left[3(\cos\theta)^2\cdot(-\sin\theta)\right]$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cancel{a}\left[\cancel{3}(\sin\theta)^{\cancel{2}}\cdot\cancel{\cos\theta}\right]}{\cancel{a}\left[\cancel{3}(\cos\theta)^{\cancel{2}}\cdot(-\cancel{\sin}\theta)\right]}$$

$$\left.\frac{dy}{dx}\right|_{at\ \theta = \frac{\pi}{4}} = \frac{\sin\theta}{-\cos\theta}$$

$\frac{\pi}{4} \rightarrow 45°$

$$= \frac{\sin(45)}{-\cos(45)}$$

$$\left.\frac{dy}{dx}\right|_{at\ \theta = \pi/4} = -1$$ //

Application of Derivative

1) Find Eq^n of Tangent & Normal

step ① : find slope of tangent

$$m_1 = \left(\frac{dy}{dx}\right)_{at\ (x_1, y_1)}$$

step ② : Find eq^n of Tangent

a) Point (x_1, y_1)

b) Slope $= m_1$

$$(y - y_1) = m_1 (x - x_1)$$

step ③ : Find slope of Normal

$$\text{slope of tangent} \times \text{slope of Normal} = -1$$

$$m_1 \times m_2 = -1$$

Step ④ Finding eq^n of Normal

1) Point (x_1, y_1)

2) Slope (m_2)

$$(y - y_1) = m_2 (x - x_1)$$

2) Radius of Curvature

step ① : Finding $\left.\frac{dy}{dx}\right|_{at\ (x_1, y_1)}$

Step ② : Finding $\left.\frac{d^2y}{dx^2}\right|_{at\ (x_1, y_1)}$

step ③

$$\text{Radius of Curvature } (\rho) = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

3) Maxima & Minima

step ① : finding $\frac{dy}{dx}$

step ② : Find value of x if $\frac{dy}{dx} = 0$

step ③ : Finding $\frac{d^2y}{dx^2}$

Step ④ finding Maxima & Minima

$\left.\frac{d^2y}{dx^2}\right|_{at\ (x_1, y_1)} =$ negative

function is Maxima

$\left.\frac{d^2y}{dx^2}\right|_{at\ (x_1, y_1)} =$ positive

function is Minima.